

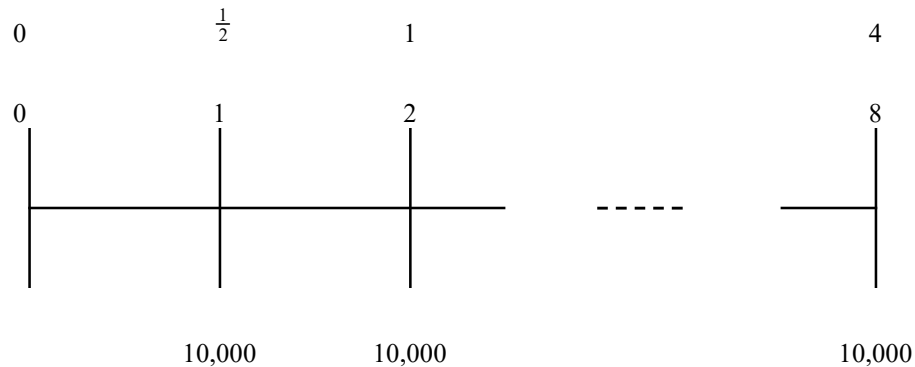
FINC 430
TA Review Session 2 Answers

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Question 1 (5-10 in the Textbook)

Your son has been accepted into college. This college guarantees that your son's tuition will not increase for the four years he attends college. The first \$10,000 tuition payment is due in six months. After that, the same payment is due every six months until you have made a total of eight payments. The college offers a bank account that allows you to withdraw money every six months and has a fixed APR of 4% (semiannual) guaranteed to remain the same over the next four years. How much money must you deposit today if you intend to make no further deposits and would like to make all the tuition payments from this account, leaving the account empty when the last payment is made?

Timeline:



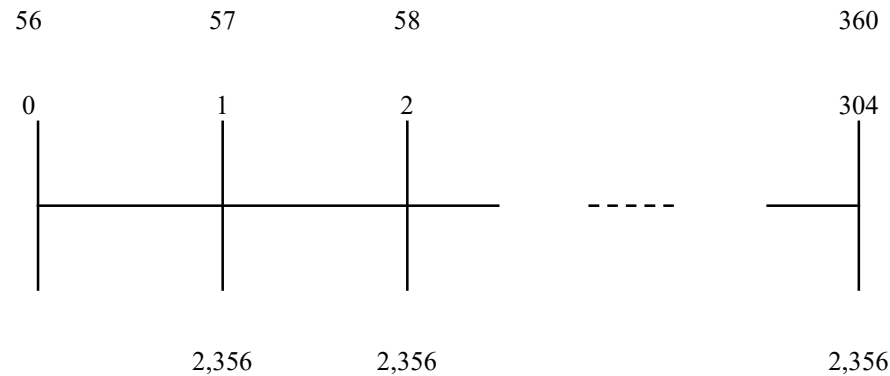
4% APR (semiannual) implies a semiannual discount rate of $\frac{4\%}{2} = 2\%$

$$\begin{aligned} \text{So, PV} &= \frac{10,000}{0.02} \left(1 - \frac{1}{(1.02)^8} \right) \\ &= \$73,254.81 \end{aligned}$$

Question 2 (5-14 in the Textbook)

You have decided to refinance your mortgage. You plan to borrow whatever is outstanding on your current mortgage. The current monthly payment is \$2356 and you have made every payment on time. The original term of the mortgage was 30 years, and the mortgage is exactly four years and eight months old. You have just made your monthly payment. The mortgage interest rate is $6 \frac{3}{8}\%$ (APR). How much do you owe on the mortgage today?

Timeline:



To find out what is owed, compute the PV of the remaining payments using the loan interest rate to compute the discount rate:

$$\text{Discount rate} = \frac{6.375}{12} = 0.53125\%$$

$$\text{So, PV} = \frac{2,356}{0.0053125} \left(1 - \frac{1}{(1.0053125)^{304}} \right)$$

$$=\$354,900$$

Question 3

Today, Suzie Minion is celebrating her 65th birthday and her retirement from her very fulfilling movie career (time 0). Due to her diligent savings, she has accumulated \$2 million in savings (at time 0). She has no other source of retirement income. To pay living expenses during her retirement, Suzie needs to withdraw from her savings account at the end of each year (first withdrawal at time 1) until her death. Suzie believes that she will live another 19 years (until age 84) and therefore plans to make 19 withdrawals from her savings (at time 1, 2, ... 19). Suppose that Suzie's retirement savings earns interest annually at a 4% nominal rate.

- (a) If Suzie plans to make equal *nominal* withdrawals, how much can she withdraw in each of the 19 years such that her savings account balance is zero after the 19th year?
- (b) Suppose instead that Suzie wanted to make equal *real* withdrawals and the inflation rate is expected to be 2.3% per year. What *real* withdrawal can Suzie make in each of the 19 years such that her savings account balance is zero after the 19th year?
- (c) According to the plan outlined in (b), how many *nominal* dollars does Suzie withdraw on her 75th birthday (time 10)?

Now suppose that Suzie is worried that due to her excellent health, she might outlive her savings by living longer than 19 additional years. So, she considers taking some of her savings and purchasing a deferred income annuity. A deferred income annuity costs money today in exchange for an annuity that begins making annual payments at some future date and continues making annual payments until you die. Prices for such products being sold by a number of insurance companies are listed in the table below.

Insurance company	Male Pays \$100,000 at Age 65 and Receives \$xx,xxx Annually for Life Starting at Age 85.	Female Pays \$100,000 at Age 65 and Receives \$xx,xxx Annually for Life Starting at Age 85.
American General	\$46,654	\$39,725
Mass Mutual	\$38,014	\$32,232
Metlife	\$45,841	\$38,056
New York Life	\$52,278	\$41,141

(d) Suppose Suzie wants to protect herself from the possibility that she will live to age 95. In such a case, Suzie would collect 11 annuity payments (at ages 85 through 95, inclusive) by purchasing a deferred income annuity today. Suppose Suzie does in fact live until 95. What rate of return does she receive if she buys a deferred income annuity from Metlife?

(e) Suzie tells her twin brother Herb about deferred income annuities and he, too, contemplates buying from Metlife. How long must Herb live to earn a higher rate of return than Suzie?

(f) Suppose that you were shopping for a deferred income annuity for your grandparents and you collected the various price quotes shown above. Suppose you had to decide in 5 seconds the company from which you would buy the annuity. Which company would you choose and why?

(a) If Suzie plans to make equal *nominal* withdrawals, how much can she withdraw in each of the 19 years such that her savings account balance is zero after the 19th year?

$$\text{EXCEL} = \text{pmt}(4\%, 19, \$2000000) = \$152,277.24.$$

(b) Suppose instead that Suzie wanted to make equal *real* withdrawals and the inflation rate is expected to be 2.3% per year. What *real* withdrawal can Suzie make in each of the 19 years such that her savings account balance is zero after the 19th year?

First, calculate the real rate of interest.

$$(1+4\%)/(1+2.3\%) - 1 = 1.662\%$$

Then repeat the above calculation with the real rate of interest to calculate the real withdrawal.

$$\text{EXCEL} = \text{pmt}(1.662\%, 19, \$2000000) = \$123,619.04.$$

(c) According to the plan outlined in (b), how many *nominal* dollars does Suzie withdraw on her 75th birthday (time 10)?

At time 10, there will have been 10 years of inflation. Therefore, the nominal amount of the withdrawal is $\$123,619.04 \times (1.023)^{10} = \$155,182.13$.

Now suppose that Suzie is worried that due to her excellent health, she might outlive her savings by living longer than 19 additional years. So, she considers taking some of her savings and purchasing a deferred income annuity. A deferred income annuity costs money today in exchange for an annuity that begins making annual payments at some future date and continues making annual payments until you die. Prices for such products being sold by a number of insurance companies are listed in the table below.

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(d) Suppose Suzie wants to protect herself from the possibility that she will live to age 95. In such a case, Suzie would collect 11 annuity payments (at ages 85 through 95, inclusive) by purchasing a deferred income annuity today. Suppose Suzie does in fact live until 95. What rate of return does she receive if she buys a deferred income annuity from Metlife?

Metlife will give Suzie \$38,056 on her 85th through 95th birthday in exchange for \$100,000 today.

In a spreadsheet, set cash flow = -\$100,000 at time 0 for the purchase of the annuity and set cash flow = \$38,056 at time 20, 21, ..., 30. Set cash flow equal to 0 for times 1-19. Use the IRR function to calculate a rate of return of 5.97%.

(e) Suzie tells her twin brother Herb about deferred income annuities and he, too, contemplates buying from Metlife. How long must Herb live to earn a higher rate of return than Suzie?

Note that Metlife will pay Herb \$45,841 a year beginning at age 85, so Metlife must believe Herb will live a shorter life than Suzie. How much less? Well, at a 5.97% rate of return, the \$100,000 investment will grow to \$300,677.91 at time 19. This amount must have the same present value as the annuity that begins at time 20 (age 85). So use the Excel function $NPER(5.97\%, -45841, 300677.91) = 8.57$. This means that if Herb collects 8.57 payments, he will earn 5.97%. The payments of the annuity are annual – that is, you can't receive a decimal part of one – so Herb has to receive 9 payments to earn more than Suzie. If Herb receives 9 payments, he must live until he reaches his 93rd birthday.

Alternatively, you could set up a time series of cash flows as you did in part (d), being sure to increase the annuity payment for Herb. Then, you could calculate IRRs for different lifespans and realize that the IRR to Herb of receiving 8 payments was 5.7%, but the IRR of receiving 9 payments was 6.1%.

(f) Suppose that you were shopping for a deferred income annuity for your grandparents and you collected the various price quotes shown above. Suppose you had to decide in 5 seconds the company from which you would buy the annuity. Which company would you choose and why?

For a fixed investment, New York Life is offering the highest annuity payment. Therefore, choose New York Life.

Question 4

You are advising a client who operates a large chain of car resale businesses on marketing their products, i.e., used cars. The client wants to determine what type of customers to target their advertising to.

You decide to analyze the choice between a typical new and used car from the vantage point of the customers. You make the following assumptions: The average time that customers own a car is 4 years for both new and used cars, so you assume that whether the customer buys a new or used car, he or she will sell it after 4 years. A new car costs \$25,000. A new car sold after 4 years sells for \$13,000. A used car costs \$13,000 and can be sold 4 years later for \$5,500. Cars bought new do not require maintenance but cars bought used require \$2,000 in maintenance at the end of each year of operation (**note: this includes the 4th year**). Apart from these differences, assume consumers think new and used cars are equivalent (i.e., they don't care about such things as the "new car smell" etc.).

(a) Determine the cash flows associated with the purchase of a new car and the cash flows associated with the purchase of a used car given the above assumptions.

(b) Consider a customer who has a low cost of capital of 5% (maybe because he or she has lots of assets or easy access to credit). Should this customer buy a new or used car?

(c) Consider now a customer who has a cost of capital of 12% (maybe because he or she has few assets or considerable other debt and hence a difficult time borrowing). What should this customer choose - new or used?

(d) Who should your client target their advertising to? Explain why the customers in part (b) and (c) make the same or different choices, as the case may be. [Hint: It may be helpful to consider the incremental cash flows of buying a used car instead of a new car to see the intuition.]

(a) Determine the cash flows associated with the purchase of a new car and the cash flows associated with the purchase of a used car given the above assumptions.

The cash flows associated with the purchase of a new and used car (in terms of purchase price, maintenance, and proceeds from resale) are:

Type	0	1	2	3	4
New	-\$25,000	0	0	0	+\$13,000
Used	-\$13,000	-\$2,000	-\$2,000	-\$2,000	-\$2,000 + \$5,500 = + \$3,500

(b) Consider a customer who has a low cost of capital of 5% (maybe because he or she has lots of assets or easy access to credit). Should this customer buy a new or used car?

Given the cost of capital of 5%, the net present value of the cost of buying a new car is

$$NPV_{new} = -\$25,000 + \frac{\$13,000}{(1 + 5\%)^4} = -\$14,305$$

whereas the net present value of the cost of buying a used car is

$$NPV_{used} = -\$13,000 - \frac{\$2,000}{5\%} \left(1 - \frac{1}{(1 + 5\%)^4} \right) + \frac{\$5,500}{(1 + 5\%)^4} \\ = -\$15,567$$

Thus, this customer should buy a new car since it is cheaper in present value terms.

(c) Consider now a customer who has a cost of capital of 12% (maybe because he or she has few assets or considerable other debt and hence a difficult time borrowing). What should this customer choose - new or used?

Given the cost of capital of 12%, the net present value of the cost of buying a new car is

$$NPV_{new} = -\$25,000 + \frac{\$13,000}{(1 + 12\%)^4} = -\$16,738$$

whereas the net present value of the cost of buying a used car is

$$NPV_{used} = -\$13,000 - \frac{\$2,000}{12\%} \left(1 - \frac{1}{(1 + 12\%)^4} \right) + \frac{\$5,500}{(1 + 12\%)^4} \\ = -\$15,579$$

Thus, this customer should buy a used car since it is cheaper in present value terms.

(d) Who should your client target their advertising to? Explain why the customers in part (b) and (c) make the same or different choices, as the case may be. [Hint: It may be helpful to consider the incremental cash flows of buying a used car instead of a new car to see the intuition.]

The client should target customers with a high cost of capital since it is in their interest to buy a used car and they are hence likely to buy. It does not make sense for customers with a low cost of capital to buy a used car, so trying to sell to them will most likely be fruitless.

The customers make different choices depending on their cost of capital. Customers with a low cost of capital buy new cars and customers with a high cost of capital buy used cars. The intuition is that by buying a used car, a customer can pay less up front and more down the road (in maintenance and reduced resale value), which is attractive to customers with a high cost of capital. To see the intuition, note that the incremental cash flows from buying a used car instead of a new car are:

Type	0	1	2	3	4
Used - New	\$12,000	-\$2,000	-\$2,000	-\$2,000	-\$9,500

Thus, buying a used car instead of a new car frees up \$12,000 at time 0, but requires extra payments of \$2,000 at time 1, 2, and 3, and of \$9,500 at time 4. This incremental cash flow has positive NPV at a cost of capital of 12% but negative NPV at a cost of capital of 5%.