FINC 430 TA Session 9
Final Review
Solutions

Marco Sammon

Topics to Review – First Half of Class

- PMT, NPV, PV and IRR functions
- Amortization tables

Borrow	Years	Rate	PMT
20000	5	5.00%	($4,619.50)
0	$ 20,000.00		
1	$16,380.50		
2	$12,580.03		
3	$8,589.54		
4	$4,399.52		
5	$0.00		

- Real vs. Nominal
  - Use real rates when working with real quantities, and nominal rates when working with nominal quantities
  - (1+nominal rate)/(1+inflation) = 1 + real rate

Quick Saving Example

- Saving for college:
  - Want 250,000 at time=18 years, will start saving at time=1, discount rate of 2%
- Can check with an amortization table

	0.02	PMT(2%,18,0,250000)
		($11,675.53)
time	to save	amt saved
0		0
1		$11,675.53
2		$23,584.56
3		$35,731.78
4		$48,121.94
5		$60,759.90
6		$73,650.63
7		$86,799.17
8		$100,210.67
9		$113,890.41
10		$127,843.75
11		$142,076.15
12		$156,593.20
13		$171,400.59
14		$186,504.12
15		$201,909.73
16		$217,623.45
17		$233,651.45
18	$250,000.00	$250,000.00

Topics to Review – Portfolio Risk and Returns

- Portfolio returns/standard deviation:
  - $E\left(r_{portfolio}\right) = xE(r_i) + (1-x)E\left(r_j\right)$
  - $\mathrm{SD}\left(r_{portfolio}\right) = \left[x^2 VAR(r_i) + (1-x)^2 VAR\left(r_j\right) + 2x(1-x)Corr\left(r_i, r_j\right) \times SD(r_i) \times SD(r_j)\right]^{\frac{1}{2}}$
- Sharpe Ratio of any portfolio: $E\left(r_p - r_f\right) / \sigma_p$

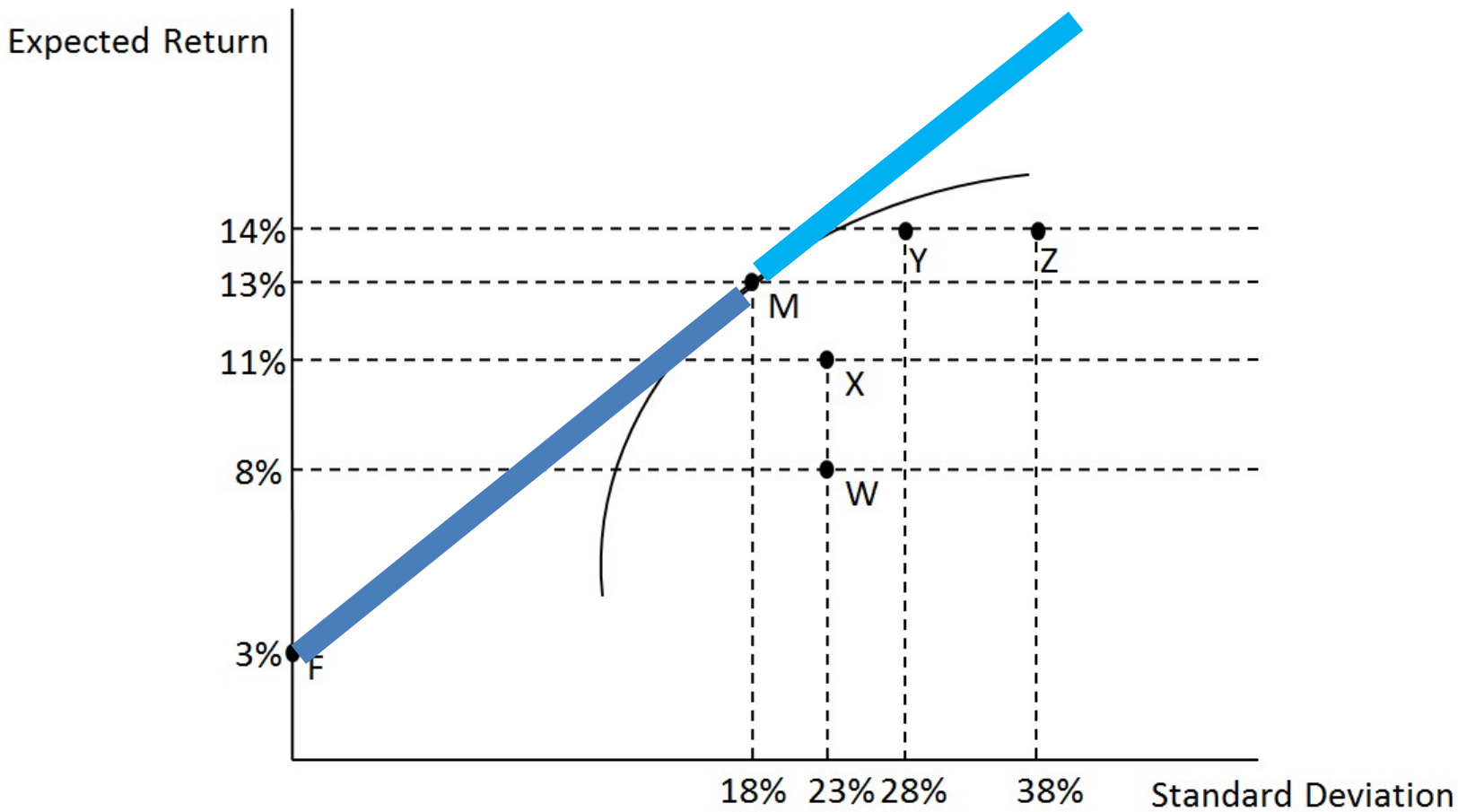
Topics to Review -- CAPM

- CAPM equation: E[R] = $r_f + \beta(E(R_m) - r_f)$
  - Given stock return, risk free rate, and market risk premium, can back out beta
- Mean-Variance Efficient Portfolios with a risk-free asset:
  - Want to find x_M, weight on market portfolio, invest (1-x_M) in the risk free asset:
  - Same mean as portfolio, same Sharpe as market
    - $E(r_p) = r_f + x_M E(r_M - r_f)$
  - Same SD as portfolio, same Sharpe as market
    - $\sigma_p = x_M \sigma_M$

Topics to Review -- MVE Portfolios

If the risk-free asset exists, mean variance efficient portfolios live on the blue line

Move up and down by applying/lowering leverage



Topics to Review – Systematic vs. Idiosyncratic Risk

- Systematic risk decomposition from the CAPM
  - Recall the CAPM **regression**:
    - R = $r_f + \beta\left(R_m - r_f\right) + idiosyncratic\ risk$
  - Apply variance operator to both sides
    - Idiosyncratic risk is uncorrelated with systematic risk
    - The variance of the risk free asset is zero
  - Var(R) = $\beta^2 Var(R_m)$ + Var(Idiosyncratic)
  - Systematic Share= $\beta^2 Var(R_m)$ / Var(R)
    - This is equal to R-squared in the CAPM regression [write proof on board]

Topics to Review – Capital Budgeting with Risk

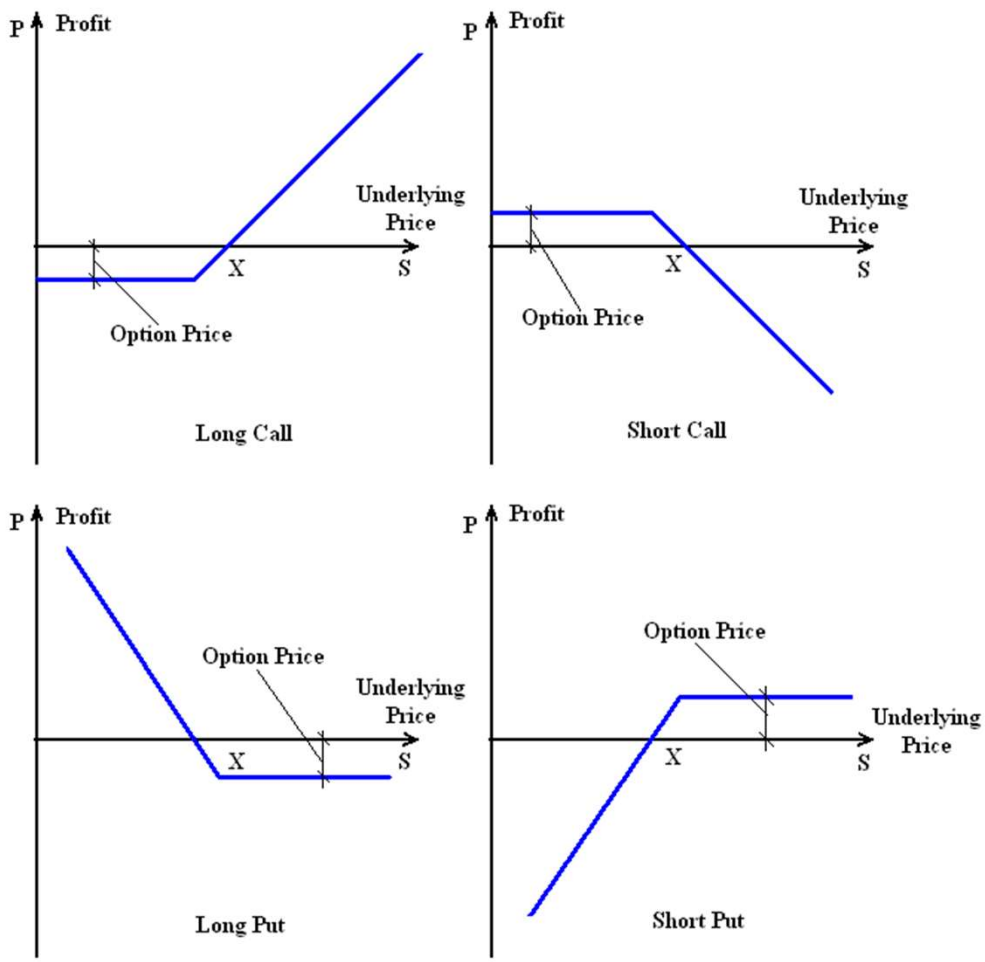
- Asset Beta (financing side): $\beta_{Assets} = \beta_{Equity}\frac{E}{V} + \beta_{Debt}\frac{D}{V}$
- Asset Beta (project side): $\beta_{Assets} = \beta_{Cash}\frac{Cash}{V} + \beta_{project\ 1}\frac{P_1}{V} + \beta_{project\ 2}\frac{P_2}{V} + \ldots$

  Remember that cash always has a beta of 0

- Calculating WACC $E(r_{Firm}) = \left(\frac{E}{V}\right)E(r_E) + \left(\frac{D}{V}\right)E(r_D)$
  - $E(r_{Debt}) = r_F + \beta_{Debt}(E(r_M) - r_F)$
  - $E(r_{Stock}) = r_F + \beta_{Stock}(E(r_M) - r_F)$

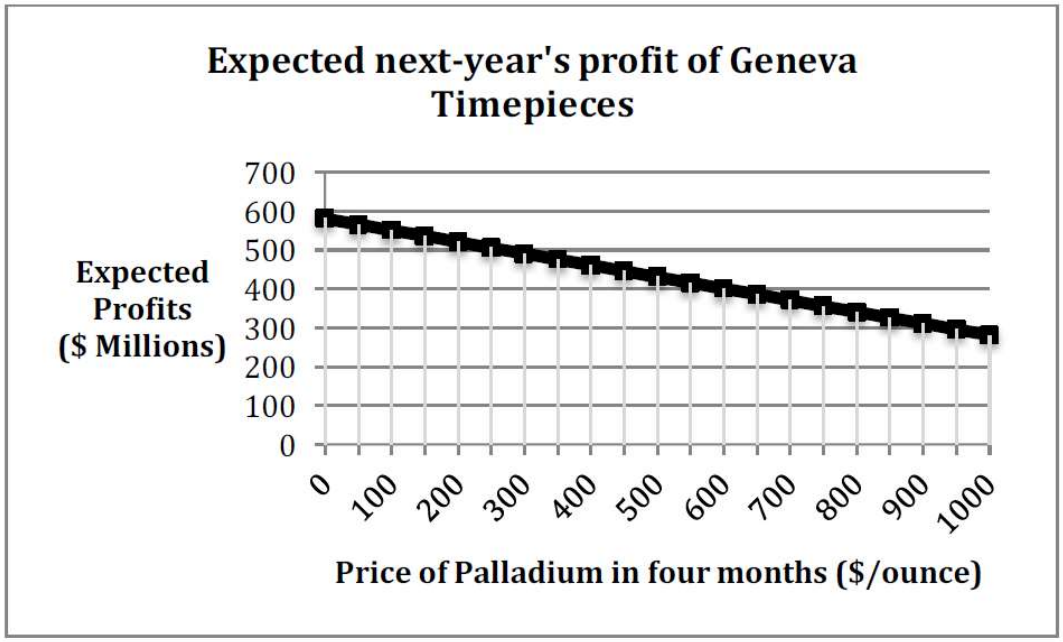
Topics to Review -- Options

- Moneyness
- Payoffs vs. Profits
- Who buys options and why?
  - Example: Producer vs. User of oil



Options: Practice Exam A Q5

You are the chief risk management officer for Geneva Timepieces, an American retailer of luxury watches from Switzerland. Through careful analysis of your company's historical data, you have determined that company profits are highly correlated with the price of palladium, an element used to make the delicate springs used in your watches. You are confident that the relationship between Geneva's expected next-year's profits and the price of palladium in four months is given by the diagram below. The diagram shows that Geneva's profits are expected to fall by $15 million for every $50 increase in the price of palladium per ounce. Palladium is currently trading at $600/ounce. If the price of palladium were to remain at its current level, the graph shows that Geneva's expected next year's profits would be $400 million.



Options: Practice Exam A Q5

You are contemplating using options to hedge the palladium price risk your firm faces. You observe the following prices for European options on palladium expiring in four months. **Each option gives you the right, but not the obligation to either buy (call) or sell (put) 100 ounces of palladium at the given strike price in four months.** (Assume these options have the appropriate expiration date for this problem.)

Strike price	Put option price	Call option price
$500.00/ounce	$714.44	$11,211.95
$550.00/ounce	$2,108.01	$7,655.27
$600.00/ounce	$3,501.58	$4,098.59
$650.00/ounce	$7,063.63	$2,710.39
$700.00/ounce	$10,923.07	$1,619.58

Options: Practice Exam A Q5

a. Draw a payoff diagram for a purchase of one call option with a strike price of 700. Label the axes and at least three points on the diagram.
b. Draw a profit diagram for a purchase of one call option with a strike price of 700. Label the axes and at least three points on the diagram.
c. Suppose you wanted to enter into an options position (involving one of the options listed above) that would guarantee that Geneva's next-year's expected profits would never fall below $430 million (not counting the cost of the options purchase). Being as specific as possible, describe the options transaction that you would make.
d. Suppose you enter into the transaction you described in (c). Does Geneva want the price of palladium to rise, fall, or stay the same between the time you enter the option position and next year? Explain.
e. Suppose you entered into the transaction you described in (c) and that palladium was trading for $750/ounce in four months. Calculate the firm's total expected profit. Be sure to distinguish the profit/loss from the options position and the profit from the firm's other activity. Ignore the time value of money.

Running a Regression

- See Excel Spreadsheet

General Advice

- Try to write something for every question on the exam
- There are good practice questions for all the review topics in:
  - Practice exams
  - TA sessions
- Go over practice exams first
  - If you have extra time, go over old TA sessions, most questions are drawn from textbook or old exams

REVIEW QUESTIONS

Question 1

Company Beer-Chile only produces and sells beer in Chile and is listed on the Chilean stock market. Beer-Chile is thinking about starting **a new business to sell toothpaste only in Indonesia.**

- Beer-Chile has debt of $20 billion Chilean pesos with a beta of 0.5, its equity value is $20 billion Chilean pesos with a beta of 1.7.

- Assume both Chile and Indonesia have financial markets that work correctly.

You have the following possible comparable companies. Each of these companies is only listed on their local stock exchange.

Company Name	Country	Industry	Equity β	Debt β	Debt-Value Ratio
Beer-Chile	Chile	Beer	1.7	0.5	0.5
Football-Beer-Chile	Chile	Beer	1.1	0.1	0.2
Clean-Toothpaste-Chile	Chile	Toothpaste	0.9	0.0	0.1
Bright-Toothpaste-Chile	Chile	Toothpaste	1.3	0.3	0.4
Cold-Beer-Indonesia	Indonesia	Beer	2.0	0.8	0.9
Good-Beer-Indonesia	Indonesia	Beer	0.6	0.0	0.1
White-Toothpaste-Indonesia	Indonesia	Toothpaste	1.1	0.3	0.2
Best-Toothpaste-Indonesia	Indonesia	Toothpaste	1.7	0.6	0. 5

a. What is the overall firm beta for Beer-Chile?

b. Which of these companies should you select as comparable firms for the new company you are starting?

c. What is the beta you should use for your new Indonesian toothpaste company?

(End of question)

a. What is the overall firm beta for Beer-Chile?

$$\beta_{Assets} = \beta_{Equity}\frac{E}{V} + \beta_{Debt}\frac{D}{V}$$

$$= 1.7(1 - 0.5) + 0.5(0.5)$$

$$= 1.1$$

b. Which of these companies should you select as comparable firms for the new company you are starting?

Select the Indoneisan toothpaste companies

c. What is the beta you should use for your new Indonesian toothpaste company?

1) Compute beta of the two comparable firms

$$\beta_{AssetsWhite-Toothpaste-In} = \beta_{Equity}\frac{E}{V} + \beta_{Debt}\frac{D}{V}$$

$$= 1.1(1 - 0.2) + 0.3(0.2)$$

$$= 0.94$$

$$\beta_{AssetsBest-Toothpaste-Ind} = \beta_{Equity}\frac{E}{V} + \beta_{Debt}\frac{D}{V}$$

$$= 1.7(1 - 0.5) + 0.6(0.5)$$

$$= 1.15$$

2) Take an average:

$$\beta_{AssetsNewCompan} = \frac{\beta_{AssetsWhite-Toothpaste-In} + \beta_{AssetsBest-Toothpaste-Indonesia}}{2}$$

$$= \frac{0.94 + 1.15}{2}$$

$$= 1.045$$

Question 2

You are the CFO of Clothing World, a retailer of clothing. All of your income comes from the sale of clothing. You expect your chain of stores to bring in free cash flow of $100 million from the clothing business in one year and you further expect this clothing-related cashflow to grow by 5% each year forever. The clothing business has a beta of 0.8. Clothing World's current Debt-to-Equity ratio is 2. You have 10 million shares of stock outstanding. Clothing World currently has an equity beta of 1.5.

For years, you have followed closely Dress and Dine Inc., another clothing retailer. Dress and Dine sells clothes with risks identical to that of Clothing World. However, Dress and Dine also **has cafes in every one of its stores**, and the sale of food at clothing stores entails **risks different from those in the clothes-selling business**. According to Dress and Dine's financial statements, you have observed that food sales account for 20% of Dress and Dine's overall business. You also observe that Dress and Dine's Debt-to-Equity ratio is 3. You calculate that Dress and Dine's equity beta is 2.0 and that Dress and Dine's debt has an expected return of 8%.

You are contemplating adding cafes to all of your clothing stores. Doing so would require an initial investment of $200 million. Beginning in one year, you expect that the new business would generate free cash flow of $10 million and you further expect the cafe business to grow at 8% forever.

Assume that the CAPM is a realistic model of the world. The expected return on the market is 15% and the risk-free rate is 5%. Ignore taxes!

a. Calculate the price per share of Clothing World before consideration of the new venture.

b. Calculate the expected return to Clothing World's debtholders before consideration of the new venture.

c. Calculate the discount rate appropriate for discounting Clothing World's expected cashflows associated with opening cafes in all of its stores.

d. Using you answer to (c), calculate the NPV of Clothing World's entry into this new line of business and decide whether Clothing World should enter this business.

(End of question)

a. Calculate the price per share of Clothing World before consideration of the new venture.

1) Calculate the fim's cost of capital using beta, rf and mrp

$$\begin{aligned} E(r_{Clothes}) &= r_F + \beta_{Clothes}(E(r_M) - r_F) \\ &= 5\% + 0.8(10\%) \\ &= 13\% \end{aligned}$$

2) Use the fim's cost of capital to calculate the value with the perp. formula

The value of the firm's assets is therefore (100 million CF, growing at 5%)

$$\frac{100}{13\% - 5\%} = \$1.25 \text{ billion}$$

A D/E ratio of 2 implies that the equity is worth 1/3 of this amount, or $416.66 million.

With 10 million shares outstanding, each share is worth $41.67.

b. Calculate the expected return to Clothing World's debtholders before consideration of the new venture.

To calculate the expected return on the debt, you need to calculate the firm's debt beta. You can solve for this from:

1) Use Asset Beta Formula to get the Debt Beta

$$\beta_{Assets(Clothes)} = \beta_{Equity}\frac{E}{V} + \beta_{Debt}\frac{D}{V}$$

$$\beta_{Debt} = \frac{\left(\beta_{Assets(Clothes)} - \beta_{Equity}\frac{E}{V}\right)}{\frac{D}{V}} = \frac{\left(0.8 - 1.5\left(\frac{1}{3}\right)\right)}{\frac{2}{3}} = 0.45$$

2) Use debt beta in the CAPM to get the expected return

$$\begin{aligned} E(r_{Debt}) &= r_F + \beta_{Debt}(E(r_M) - r_F) \\ &= 5\% + 0.45(10\%) \\ &= 9.5\% \end{aligned}$$

c. Calculate the discount rate appropriate for discounting Clothing World's expected cashflows associated with opening cafes in all of its stores.

1) Calculate Dress and Dine's asset beta:

$$\beta_{Assets(Clothes\&Cafes)}^{DressandDine} = \beta_{Equity}^{DressandDine}\frac{E}{V} + \beta_{Debt}^{DressandDine}\frac{D}{V}$$

$$\beta_{Assets(Clothes\&Cafes)}^{DressandDine} = 2(25\%) + \beta_{Debt}^{DressandDine}(75\%)$$

2) Use the CAPM to get Dress and Dine's debt beta:

$$E(r_{Debt}) = r_F + \beta_{Debt}^{DressandDine}(E(r_M) - r_F)$$

$$\beta_{Debt}^{DressandDine} = \frac{8\% - 5\%}{15\% - 5\%} = 0.3$$

3) Put this back into (1)

$$\beta_{Assets(Clothes\&Cafes)}^{DressandDine} = 2(25\%) + 0.3\ (75\%) = 0.725$$

4) Use asset beta, and revenue shares, to get the café betas. Dress and Dine has the same clothing risks as Clothing World, so it should share its asset beta of 0.8 as mentioned in the question

$$\beta_{Assets(Clothes\&Cafes)}^{DressandDine} = 0.725 = 20\%\beta_{Cafe} + 80\%\beta_{Clothing}$$

$$\beta_{Cafes} = \frac{(0.725 - 80\%(0.8))}{20\%} = 0.425$$

5) Use the café betas in the CAPM to get the expected return

$$
\begin{aligned}
E(r_{Cafes}) &= r_F + \beta_{Cafes}(E(r_M) - r_F) \\
&= 5\% + 0.425(10\%) \\
&= 9.25\%
\end{aligned}
$$

d. Using you answer to (c), calculate the NPV of Clothing World's entry into this new line of business and decide whether Clothing World should enter this business.

We can use the cost of capital from the CAPM (previous slide)

Recall: "…initial investment of $200 million. Beginning in one year, you expect that the new business would generate free cash flow of $10 million and you further expect the cafe business to grow at 8% forever."

Apply the growing perpetuity formula:

$$-200 + \frac{10}{r-8\%} = -200 + \frac{10}{9.25\%-8\%} = +\$\,600\ million$$

Since the NPV is positive, Clothing World should enter this business.

Question 4

You want to determine your optimal savings plan for retirement.

- Assume you are now 45 years old (t=0) and you will retire at the age of 65 (t=20) and you will die at the age of 90 (t=45).
- In real terms, you want to have $125,000 per year during your retirement years, from age 66 to 90.
- You will earn a nominal APR of 4% compounded monthly on your savings.
- Inflation will be 2.5% per year for your lifetime.
- There are no relevant taxes.
- You currently have no retirement savings.
- When you die there will be no money left in your savings (you should have zero savings in the bank when you die, i.e. you save just enough to live until 90 and maintain your spending throughout).
- Assume you will start saving when you are 46 (t=1) and will save the same amount each year in real terms. You will save from the age of 46 to 65 (t=20).
- Use time t=0 (age 45) as the base year to calculate all real values.

(a) How much do you need to save each year in real terms?

(b) How much cash will you have in your retirement account when you retire at the age of 65?

See Excel File for Solutions

EXTRA QUESTIONS

Question 5 - Setup

Elite Cars (ELC) is a publicly traded automobile dealership having multiple showrooms across the country. You

know the following about ELC:

- All of ELC's income currently comes from the sale/trading of automobiles.
- ELC expects to generate a Free Cash Flow of $20 Million in the coming year and expects this cash flow to grow by 5.6% each year forever.
- ELC's debt to equity ratio is 0.25 and its debt is risk free. ELC expects to maintain this debt to equity ratio forever.
- ELC has 10 million shares outstanding.
- ELC currently has an equity beta of 1.5.

Question 5 - Setup

- ELC is contemplating adding services center to each of its showrooms. Doing so would require an initial investment of $18 million. The new business is expected to generate free cash flow of $3 million a year from now (t=1) and to grow at 8.2% forever thereafter.
- Consistent with ELC's target capital structure, the service center business will also have a 0.25 debt to equity ratio. The risk associated with service operations is different from the risk of the dealership business.
- You have identified a major competitor of ELC, Fabulous Motors (FM). In addition to operating dealerships that buy and sell automobiles, FM also operates a service center in each of its showrooms. FM is valued at $100 million and in particular, its service center business is valued at $25 million and its dealership business is valued at $75 million.
- FM's dealership business has the same asset beta as ELC's dealership business.
- FM's debt to equity ratio is 3, its equity beta is 4, and its debt has an expected return of 6%.
- Assume that the CAPM is a realistic model of the world. The expected return on the market is 12% and the risk-free rate is 4%.

Question 5 - Questions

- a) Calculate the price per share of ELC before consideration of the new venture.
- b) What is the expected return to ELC's debtholders before consideration of the new venture?
- c) Calculate the discount rate appropriate for discounting ELC's expected cash flows associated with opening services centers in all of its showrooms.
- d) What will ELC's stock price be after considering the new service center business?

See Excel File for Solutions

Question 3

The price of a six-month US government zero-coupon bond is $980. The yield to maturity on a one-year US government 15% coupon bond is 4%. Assume the face value of these bonds are all $1,000. If the bond market works correctly, what should the price of a US government one-year zero-coupon bond be? Assume the face value of these bonds are all $1,000.

(End of question)

Price of 6-month zero= $980

The 6-month spot rate is $\left(\frac{1000}{980}\right)^2 - 1 = 4.12\%$

You must first determine the price of the coupon bond using the yield to maturity, which is quoted as an APR so you must divide it by 2

Coupon payment $= 1000 \times \frac{15\%}{2} = 75$

Price of the 15% coupon bond $= \frac{75}{(1+\frac{4\%}{2})} + \frac{1000+75}{(1+\frac{4\%}{2})^2} = 1{,}106.79$

Next you have to determine the one-year spot rate from the above information. This is difficult, you have to use the 6-month spot rate and the known price of the coupon bond, to back out what the one-year spot rate is.

Let the one-year spot rate be r . Then,

$$1{,}106.79 = \frac{75}{(1+4.12\%)^{\frac{1}{2}}} + \frac{1000+75}{(1+r)^1}$$

Solving, we get

$$r = 4.04\%$$

Then, given this one- year spot rate, we proceed as follows.

$$\text{Price of a one–year zero } = \frac{1000}{(1+r)^1}$$

$$= \frac{1000}{(1+4.04\%)^1}$$

$$= \$\,961.20$$